FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Tuesday 16th March, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS — TEST PAPER WITH ANSWER

SECTION-A

1. The number of elements in the set $\{x \in \mathbb{R} : (|x| - 3)\ |x + 4| = 6\}$ is equal to

(1) 3 (2) 2 (3) 4 (4) 1

Official Ans. by NTA (2)

2. Let a vector $\alpha\hat{i}+\beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\hat{i}+\hat{j}$ by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices (α, β), $(0, \beta)$ and $(0, 0)$ is equal to

(1) $\frac{1}{2}$ (2) 1

(3) $\frac{1}{\sqrt{2}}$ (4) $2\sqrt{2}$

Official Ans. by NTA (1)

3. If for $a > 0$, the feet of perpendiculars from the points $A(a, -2a, 3)$ and $B(0, 4, 5)$ on the plane $lx + my + nz = 0$ are points $C(0, -a, -1)$ and D respectively, then the length of line segment CD is equal to :

(1) $\sqrt{31}$ (2) $\sqrt{41}$

(3) $\sqrt{55}$ (4) $\sqrt{66}$

Official Ans. by NTA (4)

4. Consider three observations a, b and c such that $b = a + c$. If the standard deviation of $a + 2$, $b + 2$, $c + 2$ is d, then which of the following is true ?

(1) $b^2 = 3(a^2 + c^2) + 9d^2$
(2) $b^2 = a^2 + c^2 + 3d^2$
(3) $b^2 = 3(a^2 + c^2 + d^2)$
(4) $b^2 = 3(a^2 + c^2) - 9d^2$

Official Ans. by NTA (4)

5. If for $x \in \left(0, \frac{\pi}{2}\right)$, $\log_{10}\sin x + \log_{10}\cos x = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10}n - 1)$, $n > 0$, then the value of n is equal to :

(1) 20 (2) 12 (3) 9 (4) 16

Official Ans. by NTA (2)

6. Let $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$, $i = \sqrt{-1}$. Then, the system of linear equations $A^8\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$ has :

(1) A unique solution
(2) Infinitely many solutions
(3) No solution
(4) Exactly two solutions

Official Ans. by NTA (3)

7. If the three normals drawn to the parabola, $y^2 = 2x$ pass through the point $(a, 0)$ $a \neq 0$, then 'a' must be greater than :

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) -1 (4) 1

Official Ans. by NTA (4)

8. Let the position vectors of two points P and Q be $3\hat{i}-\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}-4\hat{k}$, respectively. Let R and S be two points such that the direction ratios of lines PR and QS are $(4, -1, 2)$ and $(-2, 1, -2)$, respectively. Let lines PR and QS intersect at T. If the vector $\overrightarrow{TA}$ is perpendicular to both $\overrightarrow{PR}$ and $\overrightarrow{QS}$ and the length of vector $\overrightarrow{TA}$ is $\sqrt{5}$ units, then the modulus of a position vector of A is :

(1) $\sqrt{482}$ (2) $\sqrt{171}$

(3) $\sqrt{5}$ (4) $\sqrt{227}$

Official Ans. by NTA (2)

9. Let the functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be defined as :

$$f(x) = \begin{cases} x+2, & x < 0 \\ x^2, & x \geq 0 \end{cases} \text{ and } g(x) = \begin{cases} x^3, & x < 1 \\ 3x-2, & x \geq 1 \end{cases}$$

Then, the number of points in $\mathbb{R}$ where $(f o g)(x)$ is NOT differentiable is equal to :

(1) 3 (2) 1
(3) 0 (4) 2

Official Ans. by NTA (2)

10. Which of the following Boolean expression is a tautology ?

(1) $(p \wedge q) \vee (p \vee q)$
(2) $(p \wedge q) \vee (p \to q)$
(3) $(p \wedge q) \wedge (p \to q)$
(4) $(p \wedge q) \to (p \to q)$

Official Ans. by NTA (4)

11. Let a complex number z, $|z| \neq 1$,

satisfy $\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right) \leq 2$. Then, the largest value of $|z|$ is equal to _____ .

(1) 8 (2) 7 (3) 6 (4) 5

Official Ans. by NTA (2)

12. If n is the number of irrational terms in the expansion of $(3^{1/4} + 5^{1/8})^{60}$, then$(n - 1)$ is divisible by :

(1) 26 (2) 30 (3) 8 (4) 7

Official Ans. by NTA (1)

13. Let P be a plane $lx + my + nz = 0$ containing the line, $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$. If plane P divides the line segment AB joining points A(–3, –6, 1) and B(2, 4, –3) in ratio k : 1 then the value of k is equal to :

(1) 1.5 (2) 3 (3) 2 (4) 4

Official Ans. by NTA (3)

14. The range of $a \in \mathbb{R}$ for which the function

$$f(x) = (4a-3)(x+\log_e 5)+2(a-7)\cot\left(\frac{x}{2}\right)\sin^2\left(\frac{x}{2}\right),$$

$x \neq 2n\pi, n \in \mathbb{N}$, has critical points, is :

(1) (–3, 1) (2) $\left[-\frac{4}{3}, 2\right]$

(3) $[1, \infty)$ (4) $(-\infty, -1]$

Official Ans. by NTA (2)

15. A pack of cards has one card missing. Two cards are drawn randomly and are found to be spades. The probability that the missing card is not a spade, is :

(1) $\frac{3}{4}$ (2) $\frac{52}{867}$

(3) $\frac{39}{50}$ (4) $\frac{22}{425}$

Official Ans. by NTA (3)

16. Let [x] denote greatest integer less than or equal to x. If for $n \in \mathbb{N}$, $(1-x+x^3)^n = \sum_{j=0}^{3n} a_j x^j$, then

$$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1}$$ is equal to :

(1) 2 (2) 2^{n-1}

(3) 1 (4) n

Official Ans. by NTA (3)

17. If $y = y(x)$ is the solution of the differential equation, $\frac{dy}{dx} + 2y\tan x = \sin x$, $y\left(\frac{\pi}{3}\right) = 0$, then the maximum value of the function y(x) over $\mathbb{R}$ is equal to :

(1) 8 (2) $\frac{1}{2}$ (3) $-\frac{15}{4}$ (4) $\frac{1}{8}$

Official Ans. by NTA (4)

18. The locus of the midpoints of the chord of the circle, $x^2 + y^2 = 25$ which is tangent to the hyperbola, $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is :

(1) $(x^2 + y^2)^2 - 16x^2 + 9y^2 = 0$
(2) $(x^2 + y^2)^2 - 9x^2 + 144y^2 = 0$
(3) $(x^2 + y^2)^2 - 9x^2 - 16y^2 = 0$
(4) $(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$

Official Ans. by NTA (4)

19. The number of roots of the equation,

$$(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$$

in the interval $[0, \pi]$ is equal to :

(1) 3 (2) 4 (3) 8 (4) 2

Official Ans. by NTA (2)

20. Let $S_k = \sum_{r=1}^{k} \tan^{-1}\left(\frac{6^r}{2^{2r+1}+3^{2r+1}}\right)$. Then $\lim_{k\to\infty} S_k$ is equal to :

(1) $\tan^{-1}\left(\frac{3}{2}\right)$ (2) $\frac{\pi}{2}$

(3) $\cot^{-1}\left(\frac{3}{2}\right)$ (4) $\tan^{-1}(3)$

Official Ans. by NTA (3)

SECTION-B

1. Consider an arithmetic series and a geometric series having four initial terms from the set {11, 8, 21, 16, 26, 32, 4}. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to ______ .

Official Ans. by NTA (3)

2. Let $f : (0, 2) \to \mathbb{R}$ be defined as

$$f(x) = \log_2\left(1+\tan\left(\frac{\pi x}{4}\right)\right).$$

Then, $\lim\limits_{n\to\infty}\frac{2}{n}\left(f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+....+f(1)\right)$ is equal to _______ .

Official Ans. by NTA (1)

3. Let ABCD be a square of side of unit length. Let a circle C_1 centered at A with unit radius is drawn. Another circle C_2 which touches C_1 and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle C_2 meet the side AB at E. If the length of EB is $\alpha+\sqrt{3}\beta$, where α, β are integers, then $\alpha + \beta$ is equal to_______.

Official Ans. by NTA (1)

4. If $\lim\limits_{x\to 0}\frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$, then a + b + c is equal to _______.

Official Ans. by NTA (4)

5. The total number of 3 × 3 matrices A having enteries from the set (0, 1, 2, 3) such that the sum of all the diagonal entries of AA^T is 9, is equal to _____.

Official Ans. by NTA (766)

6. Let

$$P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix}$$ and

$$A = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega+1 \end{bmatrix}$$

where $\omega = \frac{-1+i\sqrt{3}}{2}$, and I_3 be the identity matrix of order 3. If the determinant of the matrix $(P^{-1}AP - I_3)^2$ is $\alpha\omega^2$, then the value of α is equal to ______ .

Official Ans. by NTA (36)

7. If the normal to the curve $y(x) = \int\limits_0^x (2t^2 - 15t + 10)\,dt$ at a point (a,b) is parallel to the line x + 3y = –5, a > 1, then the value of |a + 6b| is equal to ________ .

Official Ans. by NTA (406)

8. Let the curve y = y(x) be the solution of the differential equation, $\frac{dy}{dx} = 2(x+1)$. If the numerical value of area bounded by the curve y = y(x) and x-axis is $\frac{4\sqrt{8}}{3}$, then the value of y(1) is equal to _______ .

Official Ans. by NTA (2)

9. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $f(x) + f(x + 1) = 2$, for all $x \in \mathbb{R}$. If $I_1 = \int\limits_0^8 f(x)dx$ and $I_2 = \int\limits_{-1}^3 f(x)dx$, then the value of $I_1 + 2I_2$ is equal to ________ .

Official Ans. by NTA (16)

10. Let z and w be two complex numbers such that $w = z\bar{z} - 2z + 2$, $\left|\frac{z+i}{z-3i}\right| = 1$ and Re(w) has minimum value. Then, the minimum value of $n \in \mathbb{N}$ for which w^n is real, is equal to _____.

Official Ans. by NTA (4)